Chapter 5

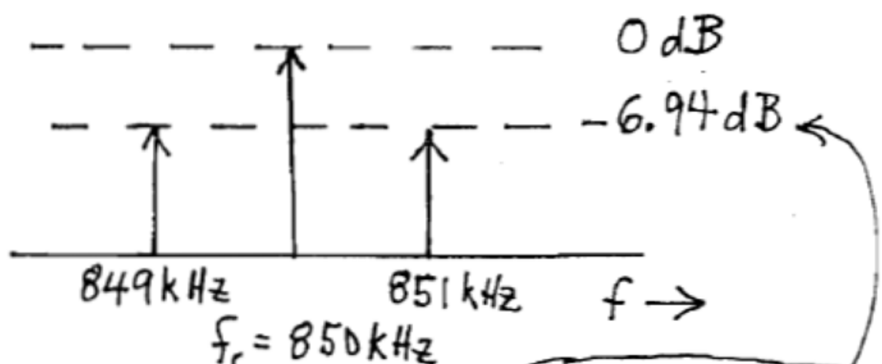
5-1.

(a.) $dBk = 10\log_{10}\left(\frac{5000}{1000}\right) = \underline{\underline{6.99\ dBk}}$

(b.) $P = \frac{A_c^2}{2R} \Rightarrow A_c = \sqrt{2PR} = \sqrt{2(5000)(50)} = \underline{\underline{707\ \text{volts}}}$

$\underline{\underline{s(t) = 707\,[1 + 0.9\cos(2000\pi t)]\,\cos(2\pi\, 850{,}000 t)}}$

(c.)
$$s(t) = 707\cos\omega_c t + \frac{0.9(707)}{2}\cos[(\omega_c - \omega_m)t] + \frac{0.9(707)}{2}\cos[(\omega_c + \omega_m)t]$$



$20\log_{10}\left(\frac{318}{707}\right) = -6.94$

(d.) $P_{AVG} = \frac{(707)^2}{2(50)} + \frac{(318)^2}{2(50)} + \frac{(318)^2}{2(50)} = \underline{\underline{7.021\ kW}}$

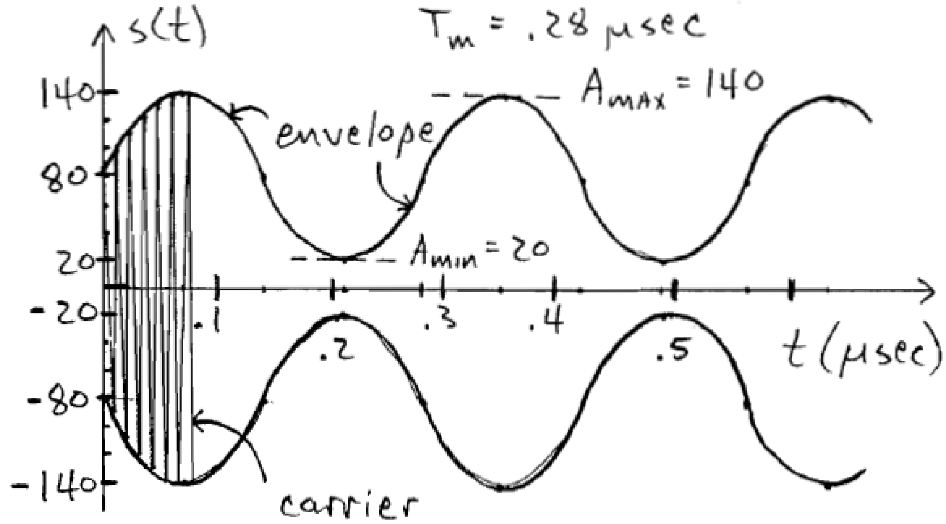
(e.) $PEP = \frac{[(707)(1.9)]^2}{2(50)} = \underline{\underline{18.045\ kW}}$

5-4. (a.) $m(t) = -0.2 + 0.6 \sin \omega_1 t$

$$f_m = f_1 = 3.57 \text{ MHz} \; ; \; A_c = \underline{\underline{100}}$$

$$s(t) = 100\left(0.8 + 0.6 \sin \omega_1 t\right) \cos \omega_c t$$

$T_m = .28\ \mu\text{sec}$

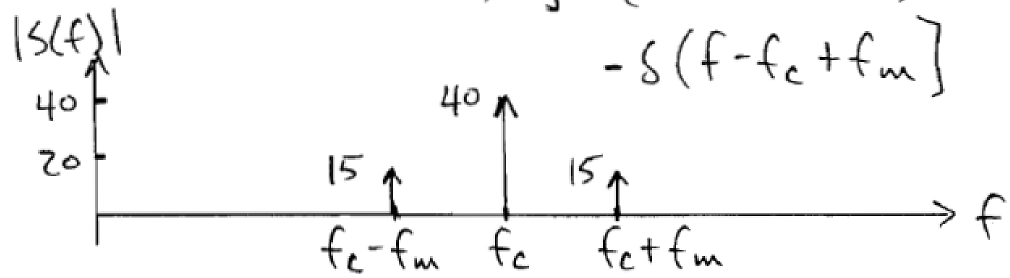


(b.) $$\% \text{ pos. mod.} = \frac{A_{max} - A_c}{A_c}(100) = \frac{140-100}{100}(100) = \underline{\underline{40\%}}$$

$$\% \text{ neg. mod.} = \frac{A_c - A_{MIN}}{A_c}(100) = \frac{100-20}{100}(100) = \underline{\underline{80\%}}$$

(c.) $\underline{f > 0}$

$$S(f) = 40\,\delta(f - f_c) - j\,15\left[\delta(f - f_c - f_m) - \delta(f - f_c + f_m)\right]$$



5-6. From (5-5a)

$$\% \text{ Pos. Mod.} = \frac{A_{max} - A_c}{A_c}(100) \overset{\text{given}}{\doteq} 120$$

where:

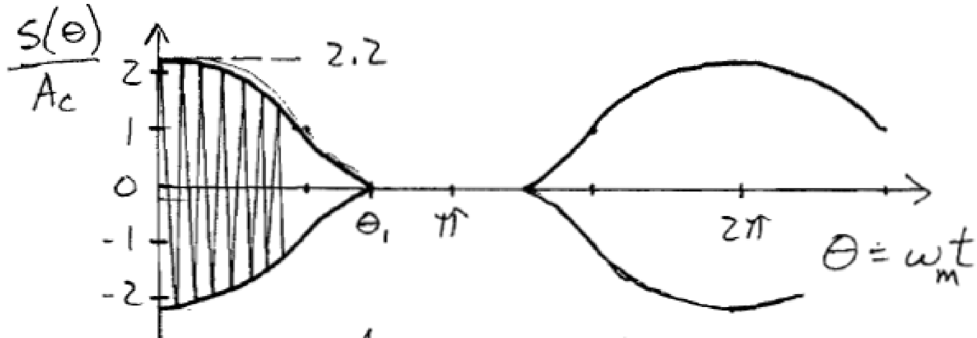
$$s(t) = \begin{cases} A_c\{1 + A_m \cos\omega_m t\}\cos\omega_c t; & m(t) \geq -1 \\ 0 & ; m(t) < -1 \end{cases}$$

$$m(t) = A_m \cos\omega_m t \quad ; \quad A_{max} = A_c[1 + A_m]$$

$$\frac{A_{MAX} - A_c}{A_c} = \underline{\underline{A_m = 1.2}}$$

$$g(t) = \begin{cases} A_c\{1 + 1.2\cos\omega_m t\}, & 1.2\cos\omega_m t \geq -1 \\ 0 & , 1.2\cos\omega_m t < -1 \end{cases}$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_m t} \Rightarrow G(f) = \sum_{-\infty}^{\infty} c_n \delta(f - nf_m)$$



$$A_m \cos\theta_1 = -1$$

Aside: $\theta_1 = \cos^{-1}\left(\frac{-1}{1.2}\right) = \underline{146.4°}$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g(t)\, e^{-jn\omega_m t}\, dt$$

$$= \frac{A_c}{2\pi}\int_{-\theta_1}^{\theta_1} [1 + A_m\cos\theta]\, e^{-jn\theta}\, d\theta$$

5-6 . Cont'd

$$c_n = \frac{A_c}{2\pi}\left[\frac{e^{-jn\theta}}{-jn}\Big|_{-\theta_1}^{\theta_1} + A_m \int_{-\theta_1}^{\theta_1} (\cos\theta)\, e^{-jn\theta}\, d\theta \right]$$

$$= \frac{A_c}{2\pi}\left[\frac{2}{n}\left(\frac{e^{jn\theta_1} - e^{-jn\theta_1}}{j2}\right) + A_m \frac{e^{-jn\theta}}{(-jn)^2+1} \right.$$

Using Sec. A-5
where $a = -jn$

$$\left. \cdot \left(-jn\cos\theta + \sin\theta\right)\Big|_{-\theta_1}^{\theta_1} \right]$$

$$= \frac{A_c}{2\pi}\left[\frac{2\sin n\theta_1}{n} + A_m \left\{ \frac{e^{-jn\theta_1}(-jn\cos\theta_1 + \sin\theta_1)}{1-n^2} - \frac{e^{jn\theta_1}(-jn\cos\theta_1 - \sin\theta_1)}{1-n^2} \right\} \right]$$

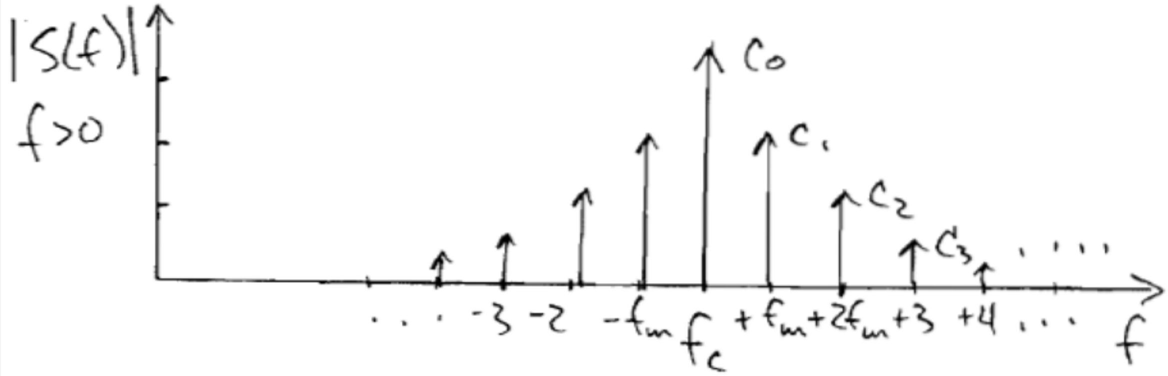
$$= \frac{A_c}{2\pi}\left[2\theta_1\left(\frac{\sin n\theta_1}{n\theta_1}\right) + A_m \left\{ \frac{jn(2j)\left(\frac{e^{jn\theta_1} - e^{-jn\theta_1}}{2j}\right)\cos\theta_1}{1-n^2} + \frac{2\left(\frac{e^{jn\theta_1} + e^{-jn\theta_1}}{2}\right)\sin\theta_1}{1-n^2} \right\} \right]$$

$A_m = 1.2$
$\theta_1 = 146.4°$

$$c_n = \frac{A_c}{2\pi}\left[2\theta_1\left(\frac{\sin(n\theta_1)}{n\theta_1}\right) + 2A_m \left\{ \frac{\cos(n\theta_1)\sin\theta_1 - n\sin(n\theta_1)\cos\theta_1}{1-n^2} \right\} \right]$$

$$S(f) = \frac{1}{2}\left[\sum_{-\infty}^{\infty} c_n \delta(f - f_c - nf_m) + \sum_{-\infty}^{\infty} c_n^* \delta(-f - f_c - nf_m) \right]$$

5-6 . Cont'd $\quad c_n = c_n^* \quad (c_n \text{ real})$



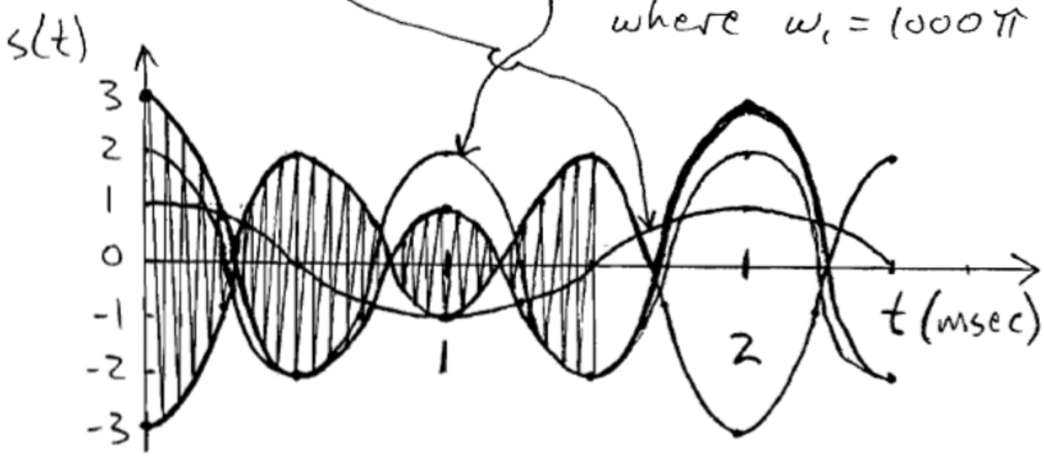
$$c_0 = \frac{A_c}{2\pi}\left[2\theta_1(\text{rad.}) + 2A_m \sin\theta_1\right]$$

$$= \frac{A_c}{2\pi}\left[2(2.56) + 2(1.2)(.553)\right]$$

5-7. (a.) DSB-SC $\quad m(t) = \cos\omega_1 t + 2\cos 2\omega_1 t$

$$s(t) = \left[\cos\omega_1 t + 2\cos 2\omega_1 t\right]\cos\omega_c t$$

where $\omega_1 = 1000\pi$

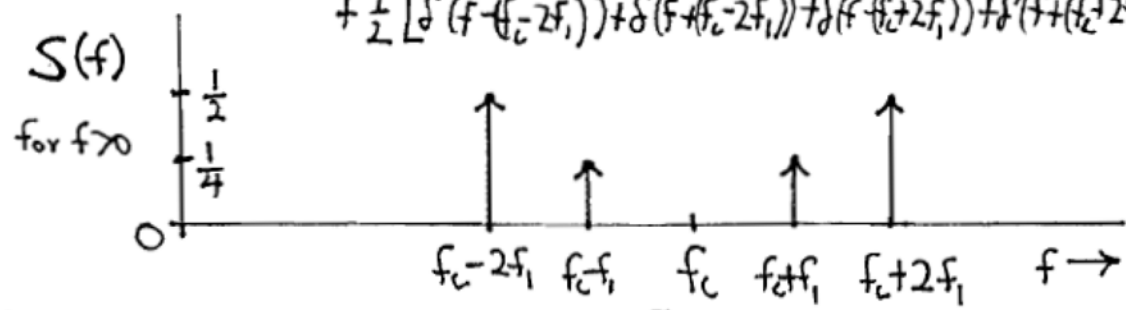


(b.) $s(t) = \frac{1}{2}\left[\cos(\omega_c - \omega_1)t + \cos(\omega_c + \omega_1)t\right]$

$$+ \cos(\omega_c - 2\omega_1)t + \cos(\omega_c + 2\omega_1)t$$

5-7 (b) Cont'd $\quad S(-f) = S(f)$ even

$$S(f) = \mathcal{F}[s(t)] = \frac{1}{4}\left[\delta(f-(f_c-f_1)) + \delta(f+(f_c-f_1)) + \delta(f-(f_c+f_1)) + \delta(f+(f_c+f_1))\right]$$
$$+ \frac{1}{2}\left[\delta(f-(f_c-2f_1)) + \delta(f+(f_c-2f_1)) + \delta(f-(f_c+2f_1)) + \delta(f+(f_c+2f_1))\right]$$



(c.) $P_{AV\,norm} = \frac{1}{2}\left[(\tfrac{1}{2})^2 + (\tfrac{1}{2})^2 + (1)^2 + (1)^2\right] = \underline{\underline{1.25\ W}}$

(d.) $A_{max} = 3 \Rightarrow PEP_{norm} = \frac{(3)^2}{2} = \underline{\underline{4.5\ W}}$

5-12. $\quad v_1(t) = \cos(2\pi(\tfrac{1}{2}B)t) = \cos(\pi B t)$

$$v_3(t) = m(t)v_1(t) = m(t)\cos(\pi B t) \leftrightarrow V_3(f) = \frac{1}{2}\left[M(f-\tfrac{1}{2}B) + M(f+\tfrac{1}{2}B)\right]$$
$$v_4(t) = m(t)v_2(t) = m(t)\sin(\pi B t) \leftrightarrow V_4(f) = \frac{1}{2}j\left[-M(f-\tfrac{1}{2}B) + M(f+\tfrac{1}{2}B)\right]$$
$$V_5(f) = \begin{cases} V_4(f), & |f| < \tfrac{1}{2}B \\ 0, & f \text{ elsewhere} \end{cases} = \begin{cases} \tfrac{1}{2}\left[M(f-\tfrac{1}{2}B) + M(f+\tfrac{1}{2}B)\right], & |f| < \tfrac{1}{2}B \\ 0, & f \text{ elsewhere} \end{cases}$$

Likewise
$$V_6(f) = \begin{cases} \tfrac{1}{2}j\left[-M(f-\tfrac{1}{2}B) + M(f+\tfrac{1}{2}B)\right], & |f| < \tfrac{1}{2}B \\ 0, & f \text{ elsewhere} \end{cases}$$

$$v_9(t) = v_5(t)\cos[2\pi(f_c+\tfrac{1}{2}B)t]$$
$$\Rightarrow V_9(f) = \frac{1}{2}\left[V_5(f-f_c-\tfrac{1}{2}B) + V_5(f+f_c+\tfrac{1}{2}B)\right]$$
$$= \frac{1}{4}\begin{cases} \left[M(f-f_c-\tfrac{1}{2}B-\tfrac{1}{2}B) + M(f-f_c-\tfrac{1}{2}B+\tfrac{1}{2}B)\right], & |f-f_c-\tfrac{1}{2}B| < \tfrac{1}{2}B \\ \left[M(f+f_c+\tfrac{1}{2}B-\tfrac{1}{2}B) + M(f+f_c+\tfrac{1}{2}B+\tfrac{1}{2}B)\right], & |f+f_c+\tfrac{1}{2}B| < \tfrac{1}{2}B \\ 0, & f \text{ elsewhere} \end{cases}$$

Aside: $|f-f_c-\frac{1}{2}B|<\frac{1}{2}B \Rightarrow -\frac{1}{2}B < f-f_c-\frac{1}{2}B<\frac{1}{2}B$

$\Rightarrow f_c+\frac{1}{2}B-\frac{1}{2}B<f<f_c+\frac{1}{2}B+\frac{1}{2}B \Rightarrow f_c<f<f_c+B$

Likewise $|f_c+f_c+\frac{1}{2}B|<\frac{1}{2}B \Rightarrow -f_c-B<f<-f_c$

Thus,

$$V_9(f)=\begin{cases}\frac{1}{4}[M(f-f_c-B)+M(f-f_c), & f_c<f<f_c+B\\ \frac{1}{4}[M(f+f_c)+M(f+f_c+B)], & -f_c-B<f<-f_c\\ 0, & f \text{ elsewhere}\end{cases}$$

Likewise

$$V_{10}(f)=\tfrac{1}{2}j\left[-V_6(f-f_c-\tfrac{1}{2}B)+V_6(f+f_c+\tfrac{1}{2}B)\right]$$

$$=\begin{cases}\frac{1}{4}[-M(f-f_c-\frac{1}{2}B-\frac{1}{2}B)+M(f-f_c-\frac{1}{2}B+\frac{1}{2}B)], & |f-f_c-\frac{1}{2}B|<\frac{1}{2}B\\ \frac{1}{4}[M(f+f_c+\frac{1}{2}B-\frac{1}{2}B)-M(f+f_c+\frac{1}{2}B+\frac{1}{2}B)], & |f+f_c+\frac{1}{2}B|<\frac{1}{2}B\\ 0, & f \text{ elsewhere}\end{cases}$$

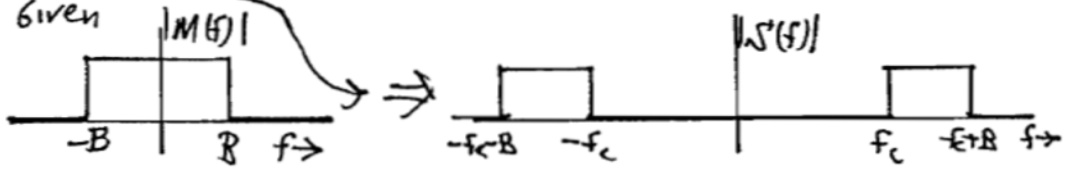
$$\Rightarrow V_{10}(f)=\begin{cases}\frac{1}{4}[-M(f-f_c-B)+M(f-f_c)], & f_c<f<f_c+B\\ \frac{1}{4}[M(f+f_c)-M(f+f_c+B)], & -f_c-B<f<-f_c\\ 0, & f \text{ elsewhere}\end{cases}$$

Output $= s(t) = v_9(t)+v_{10}(t)$

$\Rightarrow S(f)=V_9(f)+V_{10}(f)$

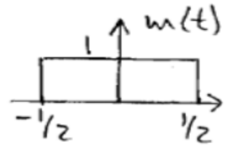
$$\Rightarrow S(f)=\begin{cases}\frac{1}{2}M(f-f_c), & f_c<f<f_c+B\\ \frac{1}{2}M(f+f_c), & -f_c-B<f<-f_c\\ 0, & f \text{ elsewhere}\end{cases} = USSB$$

Given



$\Rightarrow s(t)$ is a USSB signal QED

5-14. $m(t) = \begin{cases} 1, & |t| < 1/2 \\ 0, & t \text{ elsewhere} \end{cases}$



(a.) $\hat{m}(t) = m(t) * \frac{1}{\pi t}$

$$= \int_{-1/2}^{1/2} \frac{1}{\pi}\frac{1}{t-\lambda}\, d\lambda = \frac{-1}{\pi}\int_{t+1/2}^{t-1/2} \frac{1}{\lambda_1}\, d\lambda_1$$

$\lambda_1 = t - \lambda$
$d\lambda_1 = -d\lambda$

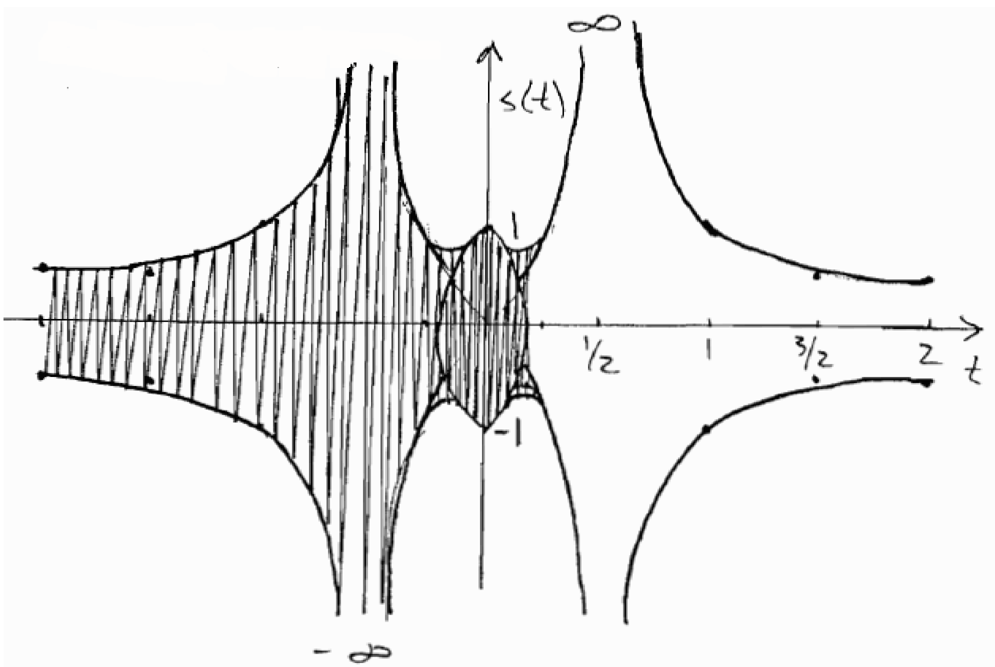
$$= \int_{t-1/2}^{t+1/2} \frac{1}{\pi}\frac{1}{\lambda_1}\, d\lambda_1 = \frac{1}{\pi}\left(\ln|\lambda_1|\right)\Big|_{t-1/2}^{t+1/2}$$

$$\hat{m}(t) = \frac{1}{\pi}\ln\left[\frac{|t+\frac{1}{2}|}{|t-\frac{1}{2}|}\right]$$

(b.) For USSB:

$$s(t) = m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t$$

$$= \Pi(t)\cos\omega_c t - \frac{1}{\pi}\ln\left[\frac{|t+1/2|}{|t-1/2|}\right]\sin\omega_c t$$



(c.) $s(t)_{max} = \infty$

5-16. Note: T has units of Hz.

(a.)
$$m(t)=\frac{\sin(\pi T t)}{\pi T t} \longleftrightarrow M(f)=\frac{1}{T}\Pi\left(\frac{f}{T}\right)=\frac{1}{T}\left[\Pi\left(\frac{f-\frac{T}{4}}{T/2}\right)+\Pi\left(\frac{f+\frac{T}{4}}{T/2}\right)\right]$$

$$\Rightarrow \mathcal{F}[\hat{m}(t)]=M(f)\begin{Bmatrix}-j, & f>0\\ j, & f<0\end{Bmatrix}=\frac{\frac{1}{2}}{\frac{T}{2}}\left[-j\Pi\left(\frac{f-T/4}{T/2}\right)+j\Pi\left(\frac{f+T/4}{T/2}\right)\right]$$

$$\Rightarrow \hat{m}(t)=-j\frac{1}{2}\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}e^{j2\pi\frac{T}{4}t}+j\frac{1}{2}\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}e^{-j2\pi\frac{T}{4}t}$$

$$=\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}\,\frac{e^{j\pi\frac{T}{2}t}-e^{-j\pi\frac{T}{2}t}}{2j}=\frac{\sin(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}\sin(\pi\tfrac{T}{2}t)=\frac{\sin^2(\pi\frac{T}{2}t)}{\pi\frac{T}{2}t}$$

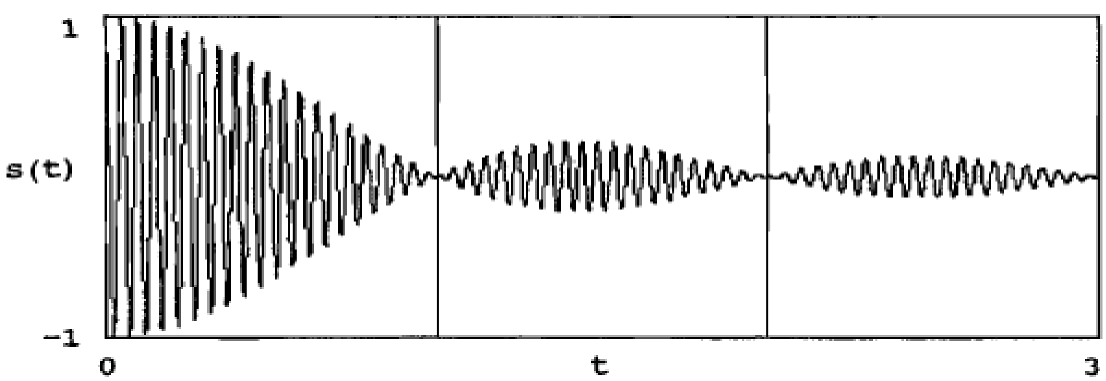
(b)

$t := 10^{-7}, 0.002 .. 3$ $\qquad T := 2$

$f_c := 20$ $\qquad \omega_c := 2\cdot\pi\cdot f_c$

$$m(t) := \frac{\sin(\pi\cdot T\cdot t)}{\pi\cdot T\, t} \qquad m_h(t) := \frac{\left[\sin\left[\pi\cdot\frac{T}{2}\, t\right]\right]^2}{\pi\cdot\frac{T}{2}\, t}$$

$$s(t) := m(t)\cdot\cos\left[\omega_c\cdot t\right] - m_h(t)\cdot\sin\left[\omega_c\cdot t\right]$$



5-21.

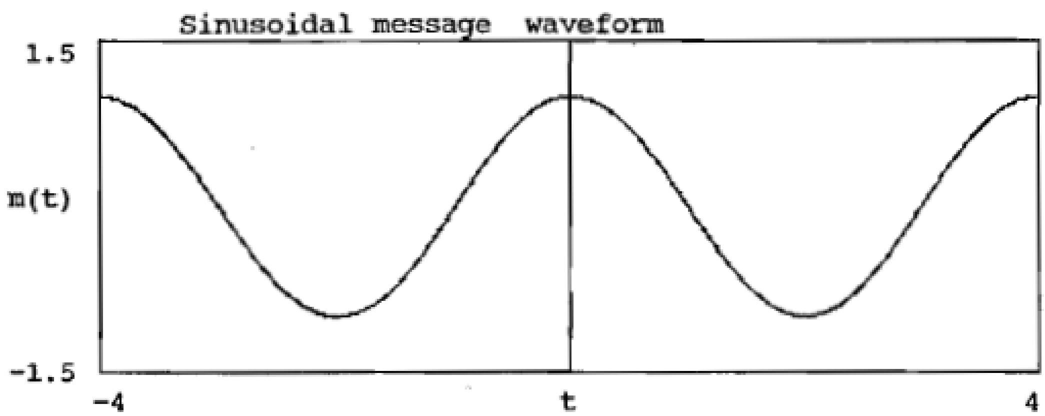
$t := -5, -4.99 \,..\, 5 \qquad fc := 1 \quad fm := 0.25\, fc \qquad Dp := \pi \quad Df := \pi$

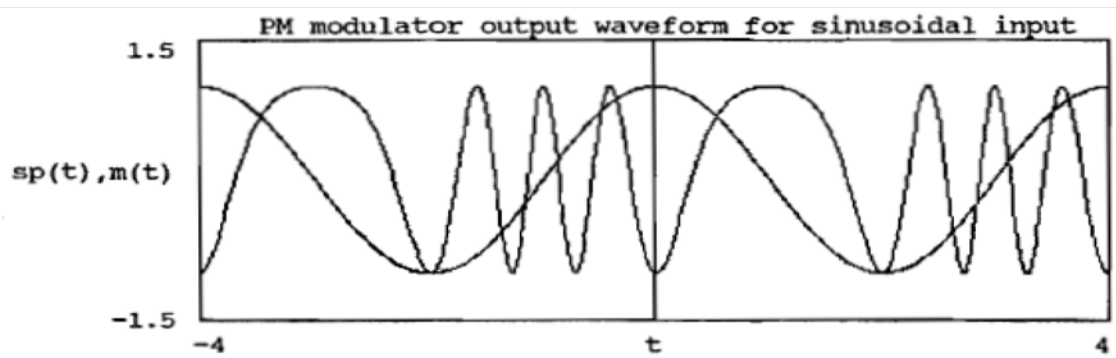
$m(t) := \cos(2\cdot\pi\ fm\cdot t)$

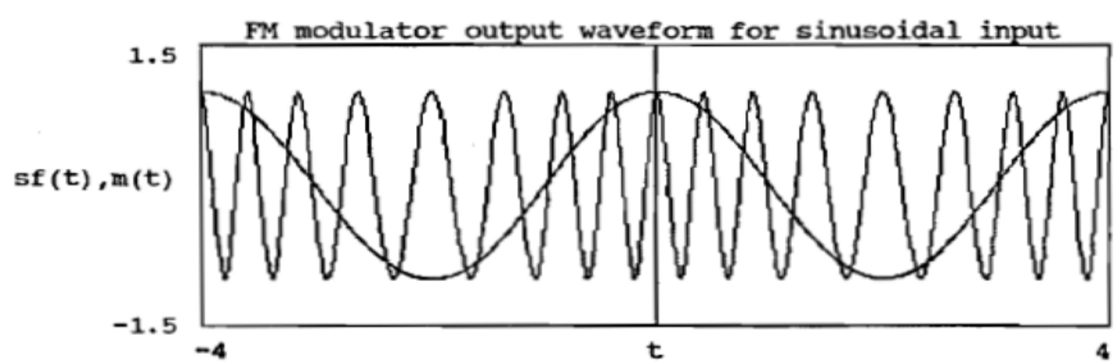
$$\theta(t) := \left[\frac{Df}{2\cdot\pi\cdot fm}\right]\cdot\sin(2\ \pi\ fm\cdot t)$$

$sp(t) := \cos(2\ \pi\cdot fc\cdot t + Dp\cdot m(t))$

$sf(t) := \cos(2\cdot\pi\cdot 2\cdot fc\cdot t + \theta(t))$







5-24. (a.) $f_{BPF} = \dfrac{103.7}{8}\ \text{MHz} = \underline{\underline{12.96\ \text{MHz}}}$

$$\Delta F_{BPF} = \frac{75\ \text{KHz}}{8} = 9.375\ \text{KHz}$$

$$BW_{BPF} = 2(\Delta F + f_m) = 2(9.375 + 15)\ \text{KHz}$$
$$= \underline{\underline{48.75\ \text{KHz}}}$$

(b.) $f_{BPF} = f_c + f_o \Rightarrow f_o = 12.96 - 5 = \underline{\underline{7.96\ \text{MHz}}}$

$f_{BPF} = f_c - f_o \Rightarrow f_o = 12.96 + 5 = \underline{\underline{17.96\ \text{MHz}}}$

$f_c = 5\ \text{MHz}$

(c.) $\Delta F_{FME} = \dfrac{75\ \text{KHz}}{8} = \underline{\underline{9.38\ \text{KHz}}}$

5-26. (a.) $\theta(t) = D_p m_p(t) = 20 \cos \omega_1 t$

$$\Rightarrow m_p(t) = \frac{20}{D_p} \cos \omega_1 t = \underline{\underline{0.2 \cos(2000\pi t}}$$

$$m_p(t)_{peak} = \underline{\underline{0.2\ V}} \quad ; \quad f_m = \underline{\underline{1\ KHz}}$$

(b.) $\theta(t) = D_f \int_{-\infty}^{t} m_f(\lambda)\, d\lambda = 20 \cos \omega_1 t$

$$\Rightarrow m_f(t) = \frac{20}{D_f} \frac{d}{dt}\left[\cos \omega_1 t\right]$$

$$= \frac{-20}{10^6}(2000\pi) \sin \omega_1 t$$

$$m_f(t) = \underline{\underline{-.1257 \sin \omega_1 t}}$$

$$m_f(t)_{peak} = \underline{\underline{.1257\ V}} \quad ; \quad f_m = \underline{\underline{1\ KHz}}$$

(c.) $P_{AV} = \frac{V_{rms}^2}{R} = \frac{(500)^2}{2(50)} = \underline{\underline{2.5\ KW}}$

$$PEP = \underline{\underline{2.5\ KW}}$$

5-30. (a.) $P_T = \frac{1}{2}\langle |g(t)|^2 \rangle = \frac{A_c^2}{2} = \frac{(100)^2}{2} = \underline{\underline{5{,}000 \text{ watts}}}$

(b.) $P_T = \frac{1}{2}\langle |g(t)|^2 \rangle \underset{\text{Using (5-55)}}{=} \frac{1}{2}\left\langle \sum_{n=-N}^{N} |c_n|^2 \right\rangle = \frac{1}{2}\sum_{n=-N}^{N} |c_n|^2 \underset{\text{Using (5-57)}}{=} \frac{1}{2} A_c^2 \sum_{n=-N}^{N} J_n(\beta)$

$$\Rightarrow P_T = \frac{1}{2} A_c^2 \left[J_0^2(\beta) + 2\sum_{n=1}^{N} J_n^2(\beta) \right]$$

where $2Nf_m = B_T \Rightarrow N = \frac{B_T}{2f_m} = \frac{56 \text{ Hz}}{2(8)} = 3.5 \Rightarrow$ Use $N=3$

Also, $\beta = \frac{\Delta F}{f_m} = \frac{k_d A_m}{f_m} = \frac{(8 \text{ Hz/volt})(5 \text{ volt})}{8 \text{ Hz}} = 5.0$

Using MathCAD: Ac := 100 β := 5.0 N := 3 n := 1 ..N

$$PT := 0.5 \cdot Ac^2 \left[J0(\beta)^2 + 2 \cdot \sum_n Jn(n,\beta)^2 \right]$$

$\underline{PT = 2583.485 \text{ watts}}$ (normalized for 1Ω)

$\boxed{5\text{-}31.}$ $s(t) = A_c \cos[w_c t + \beta \sin w_m t]$

where $A_c = 1$, $w_c = 2\pi(146.52 \times 10^6)$

$w_m = 2\pi(10^3)$, $\beta = 45^\circ \left(\frac{\pi \text{ radians}}{180^\circ}\right) = 0.7854 \text{ rad.}$

(5-60) $G(f) = \sum_{n=-\infty}^{\infty} J_n(\beta)\,\delta(f - nf_m)$

5-31 . Cont'd

$\delta(-f) = \delta(f)$

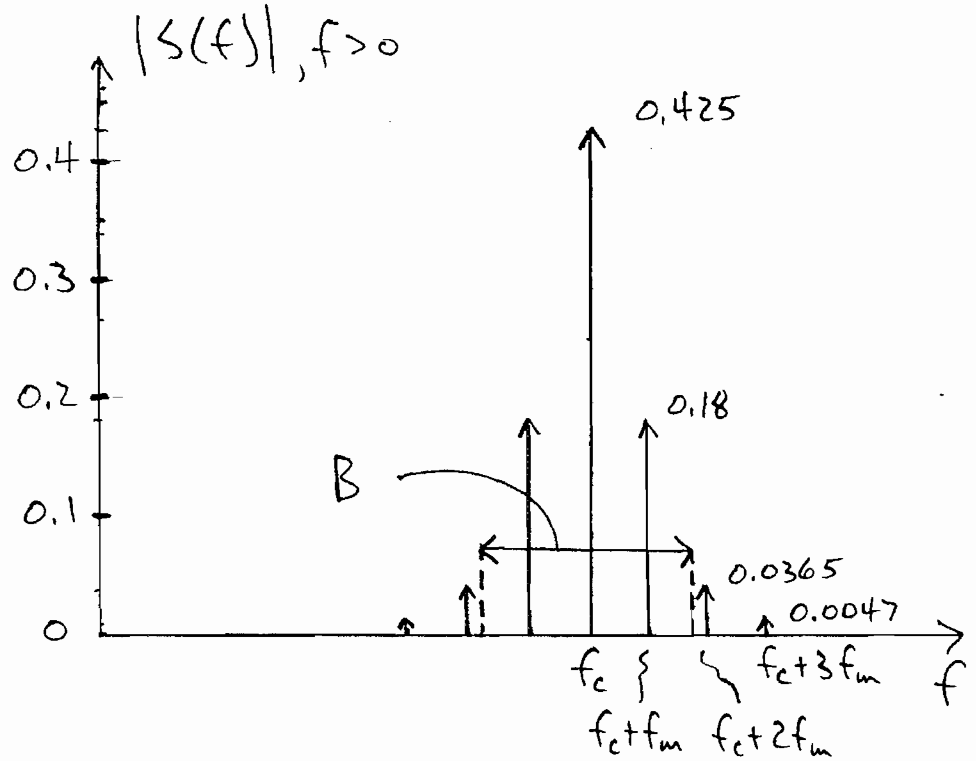
$$S(f) = \frac{1}{2}\left[G(f-f_c) + G^*(-f-f_c)\right]$$

$$= \frac{1}{2}\left\{\sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f-nf_m-f_c) + \delta(f-nf_m+f_c)\right]\right\}$$

n	$J_n(\beta)$
0	0.85
1	0.36
2	0.073
3	0.0094

Carson's Rule BW:

$$B = 2f_m(\beta+1)$$
$$= 2(1\,KHz)(1.7854)$$
$$= 3.57\,KHz$$



$$\text{Total } P_{AV} = \frac{A_c^2}{2} = 0.5$$

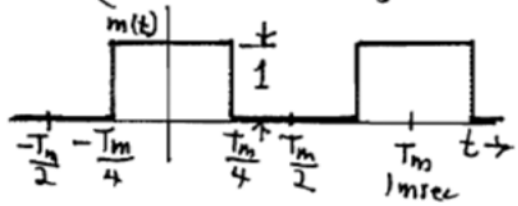
$$P_{av}(\text{within } B) = \frac{(.85)^2 + 2(.36)^2}{2} = \frac{0.9817}{2} = 98.17\%$$

5-35 $s(t) = \mathrm{Re}\{g(t)\, e^{j\omega_c t}\} = \mathrm{Re}\{10\, e^{j\theta(t)}\, e^{j\omega_c t}\}$

$\theta(t) = \beta m(t)$

$\beta = 45^\circ \left(\frac{\pi \text{ rad}}{180^\circ}\right) = 0.785 = \pi/4$

$g(t) = \sum_{-\infty}^{\infty} c_n e^{jn\omega_m t}$



$$c_n = \frac{1}{T_m}\int_{-T_m/2}^{T_m/2} g(t)\, e^{-jn\omega_m t}\, dt = \frac{10}{T_m}\left[\int_{-T_m/2}^{-T_m/4} e^{j0} e^{-jn\omega_m t}\, dt + \int_{-T_m/4}^{T_m/4} e^{j\beta} e^{-jn\omega_m t}\, dt + \int_{T_m/4}^{T_m/2} e^{j0} e^{-jn\omega_m t}\, dt\right]$$

$$c_n = \frac{10}{T_m}\left[\int_{T_m/4}^{T_m/2} e^{jn\omega_m t}\, dt + \int_{-T_m/4}^{T_m/4} e^{j\beta} e^{-jn\omega_m t}\, dt + \int_{T_m/4}^{T_m/2} e^{-jn\omega_m t}\, dt\right]$$

$$= \frac{20}{T_m}\int_{T_m/4}^{T_m/2} \underbrace{\left(\frac{e^{jn\omega_m t} + e^{-jn\omega_m t}}{2}\right)}_{\cos n\omega_m t} dt + \frac{10 e^{j\beta}}{T_m}\left.\frac{e^{-jn\omega_m t}}{-jn\omega_m}\right|_{-\frac{T_m}{4}}^{\frac{T_m}{4}}$$

$$= \frac{20}{T_m}\left[\left.\frac{\sin(n\omega_m t)}{n\omega_m}\right|_{T_m/4}^{T_m/2} + e^{j\beta}\,\frac{e^{j\pi/2} - e^{-j\pi/2}}{2j\, n\omega_m}\right]$$

$$= \frac{20}{T_m}\left[\frac{\overset{0}{\cancel{\sin(n\pi)}} - \sin\left(\frac{n\pi}{2}\right)}{n\omega_m} + e^{j\beta}\,\frac{\sin\left(\frac{n\pi}{2}\right)}{n\omega_m}\right]$$

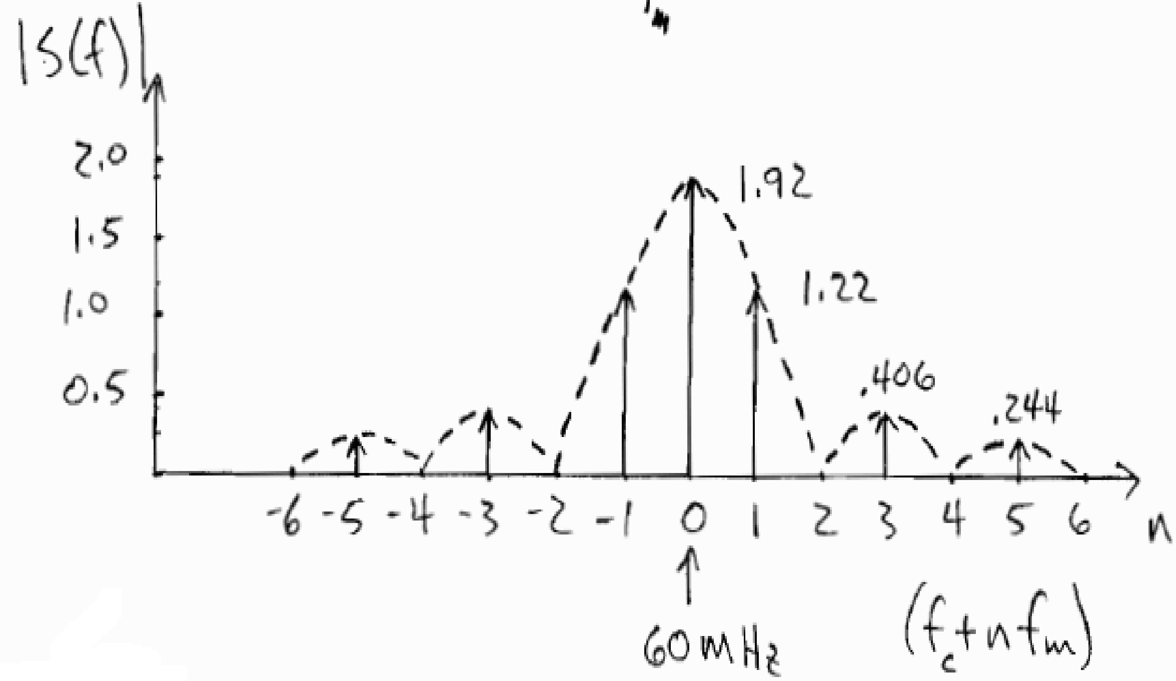
$$c_n = 5\left(e^{j\beta} - 1\right)\left[\frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2}}\right], \quad \beta = \pi/4$$

$$|c_n| = 5\sqrt{(\cos\beta - 1)^2 + (\sin\beta)^2}\;\left|\frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2}}\right|$$

$$\Rightarrow |c_n| = 3.83\left|\frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2}}\right|$$

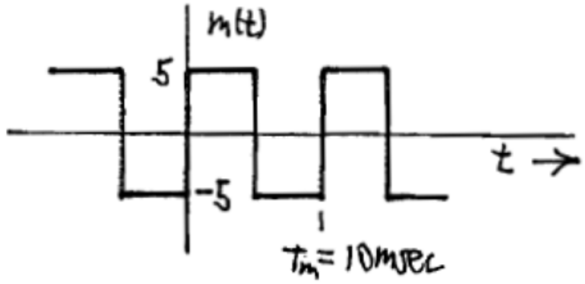
$$S(f) = \frac{1}{2}\left[\sum_{-\infty}^{\infty} c_n\, \delta(f - f_c - nf_m) + \sum_{-\infty}^{\infty} c_n^*\, \delta(f + f_c + nf_m)\right]$$

5-35. Cont'd. $f_m = 1\text{ kHz} = \frac{1}{T_m}$



5-39. NBFM

$$\theta(t) = D_f \int^t m(\lambda)\,d\lambda$$



(a.)

$$\Delta\theta = D_f \int_0^{T_m/2} m(t)\,dt = D_f \int_0^{T_m/2} 5\,dt = D_f\, 5t \Big|_0^{T_m/2} = D_f\, 5\,\frac{T_m}{2} \underset{\text{Set}}{=} (10^\circ)\left(\frac{\pi\,\text{rad}}{180^\circ}\right)$$

$$\Rightarrow D_f = \frac{10\pi}{180}\left(\frac{2}{5T_m}\right) = \frac{20\pi}{5(180)10^{-2}} = \underline{\underline{6.98\ \frac{\text{rad}}{\text{V}\cdot\text{sec}}}}$$

$$(5\text{-}6) \Rightarrow \Delta F = \frac{D_f V_p}{2\pi} = \frac{6.98(5)}{2\pi} = \underline{\underline{5.55\ \text{Hz}}}$$

(b.) From (5-26) and (5-27)

$$S'(f) = \frac{A_c}{2}\Big\{\delta(f-f_c) + \delta(f+f_c) + \frac{D_f}{2\pi}\,\frac{M(f-f_c)}{f-f_c} - \frac{D_f}{2\pi}\,\frac{M(f+f_c)}{f+f_c}$$

$$M(f) = \mathcal{F}[m(t)] = \sum_{-\infty}^{\infty} c_n\,\delta(f - nf_m) \quad \text{where } f_m = \frac{1}{10\,\text{msec}} = \underline{\underline{100\ \text{Hz}}}$$

$$S'(f) = \frac{A_c}{2}\Big\{\delta(f-f_c) + \delta(f+f_c) + \frac{D_f}{2\pi}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\left(\frac{c_n}{nf_m}\right)\Big[\delta(f-f_c-nf_m) - \delta(f+f_c-nf_m)\Big]\Big\}$$

5-39. (b.) Cont'd. Aside: Evaluate c_n

$$c_n = \frac{1}{T_m}\int_0^{T_m} m(t)\, e^{-jn\omega_m t}\, dt$$

$$= \frac{1}{T_m}\left[\int_0^{T_m/2} 5 e^{-jn\omega_m t}\, dt + \int_{T_m/2}^{T_m} (-5)\, e^{-jn\omega_m t}\, dt\right]$$

$$= \frac{5}{T_m}\left[\left.\frac{e^{-jn\omega_m t}}{-jn\omega_m}\right|_0^{T_m/2} - \left.\frac{e^{-jn\omega_m t}}{-jn\omega_m}\right|_{T_m/2}^{T_m}\right]$$

$$= \frac{5}{T_m}\left[\frac{e^{-jn\frac{\omega_m T_m}{2}} - e^{-j0} - e^{-jn\omega_m T_m} + e^{-jn\frac{\omega_m T_m}{2}}}{-jn\omega_m}\right]$$

$$= \frac{5}{T_m}\left[\frac{2e^{-jn\pi} - 1 - \overset{1}{\cancel{e^{-jn2\pi}}}}{-jn\omega_m}\right]$$

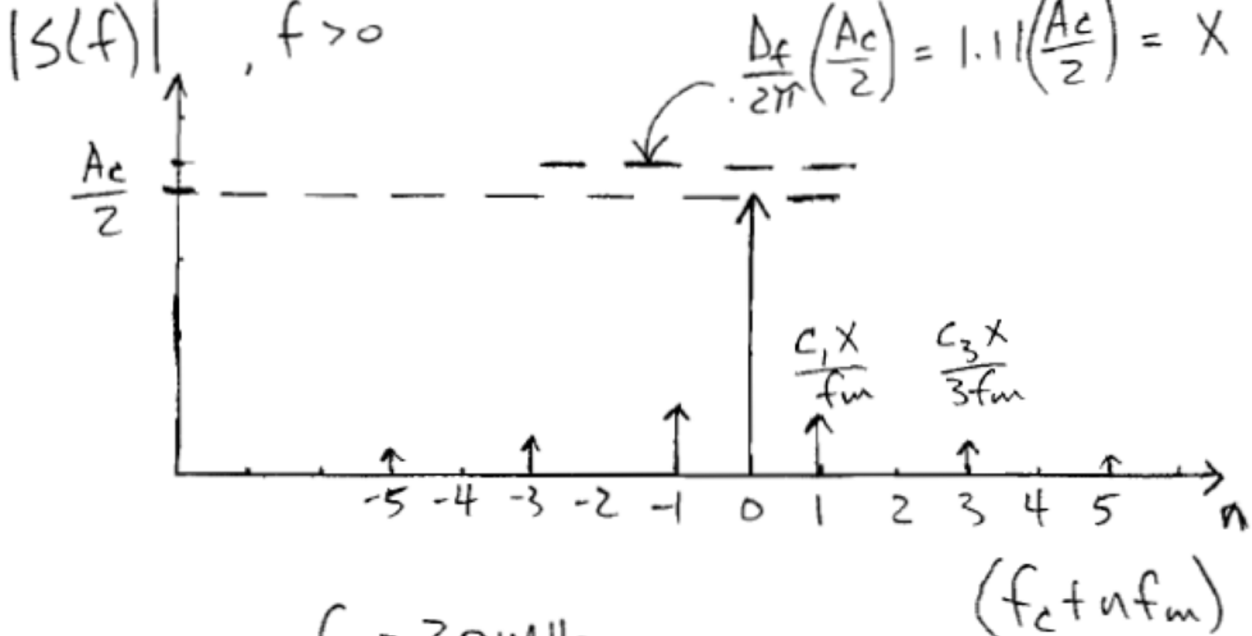
$$= 10\left[\frac{1 - e^{-jn\pi}}{jn\omega_m T_m}\right] = 10\, e^{-j\frac{n\pi}{2}}\left[\frac{e^{j\frac{n\pi}{2}} - e^{-jn\frac{\pi}{2}}}{j2\pi n}\right]$$

$$= \frac{10}{2}\, e^{-j\frac{n\pi}{2}}\left[\frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi/2}\right]$$

$$c_n = 5\,(-j)^n\left[\frac{\sin n\pi/2}{n\pi/2}\right],\ n \neq 0\ ;\ c_0 = 0$$

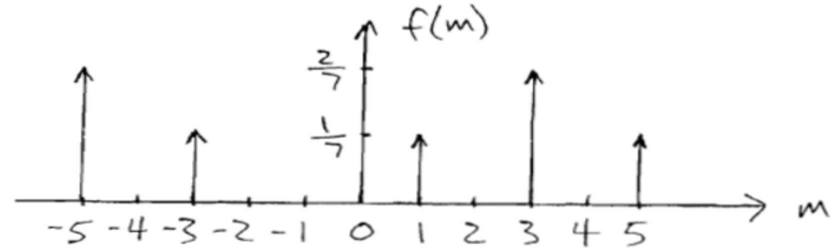
5-39. (b.) Cont'd

$C_n = 0$, n = even



$f_c = 30\,\text{MHz}$

5-42. $A_c = 5$, $f_c = 2\,\text{GHz}$, $\Delta_f = 10^5$



$$(4\text{-}131) \Rightarrow \quad \mathcal{P}(f) = \frac{\pi A_c^2}{2D_f}\left[f\left(\frac{2\pi}{D_f}(f-f_c)\right) + f\left(\frac{2\pi}{D_f}(-f-f_c)\right)\right]$$

$$\delta(ax) = \frac{1}{|a|}\delta(x), \quad a = \frac{2\pi}{D_f}\ ; \ \delta(-x) = \delta(x)$$

$$= \frac{A_c^2}{4}\Big\{\frac{2}{7}\delta(f-f_c+5f_0) + \frac{1}{7}\delta(f-f_c+3f_0)$$

$$+ \frac{1}{7}\delta(f-f_c-f_0) + \frac{2}{7}\delta(f-f_c-3f_0)$$

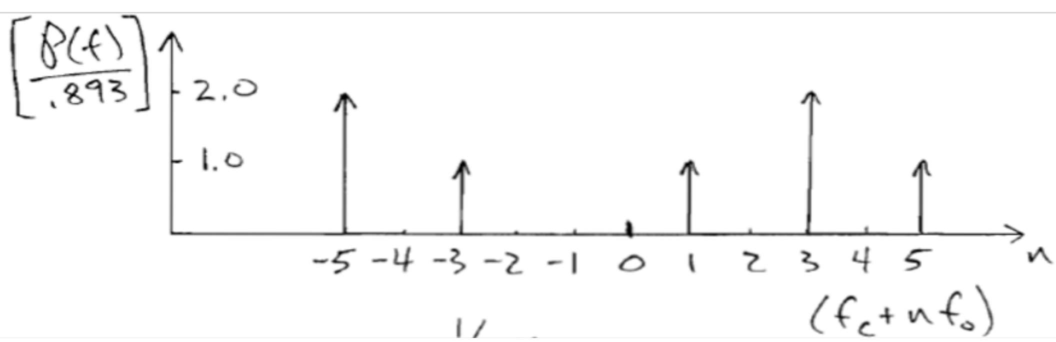
$$+ \frac{1}{7}\delta(f-f_c-5f_0) + \frac{2}{7}\delta(f+f_c-5f_0)$$

$$+ \frac{1}{7}\delta(f+f_c-3f_0) + \frac{1}{7}\delta(f+f_c+f_0)$$

$$+ \frac{2}{7}\delta(f+f_c+3f_0) + \frac{1}{7}\delta(f+f_c+5f_0)\Big\}$$

where $f_0 = \frac{D_f}{2\pi} = 15.9\,\text{kHz}$

$$\frac{A_c^2}{4(7)} = \frac{25}{28} = 0.893$$



5-45. (a.) Assuming the BW's are absolute BW's :

The BW of $s(t)$ is $2B_2 = \underline{\underline{20 \text{ KHz}}}$

$$s(t) = m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t$$

$$= Re\left\{\left[m_1(t) - j\,m_2(t)\right] e^{j\omega_c t}\right\}$$

$$g(t) = m_1(t) - j\,m_2(t)$$

$$G(f) = M_1(f) - j\,M_2(f)$$

$$S(f) = \frac{1}{2}\left[G(f-f_c) + G^*(-f-f_c)\right]$$

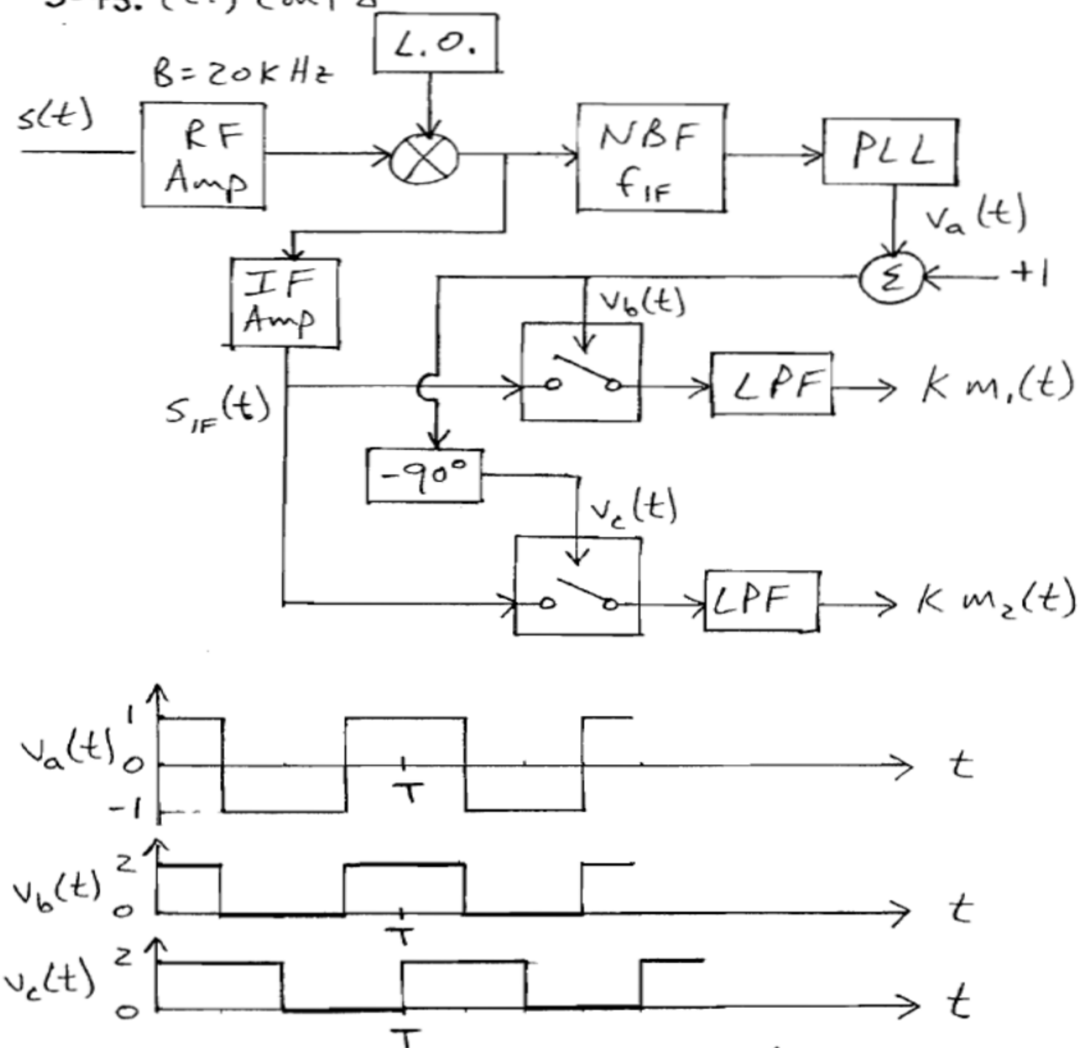
$$= \frac{1}{2}\Big[M_1(f-f_c) + M_1^*(-f-f_c) - j\,M_2(f-f_c) + j\,M_2^*(-f-f_c)\Big]$$

for $m(t)$ real : $M^*(-f) = M(f)$

$$= \frac{1}{2}\left[M_1(f-f_c) + M_1(f+f_c)\right] - \frac{j}{2}\left[M_2(f-f_c) - M_2(f+f_c)\right]$$

(c.) Assume $m_1(t)$ has a constant D.C. value, therefore providing a discrete <u>carrier</u> term.

5-45. (c.) Cont'd



$$T = \frac{1}{f_{IF}}$$

$s_{IF}(t)$ is sampled once every T seconds by <u>each</u> switch, therefore

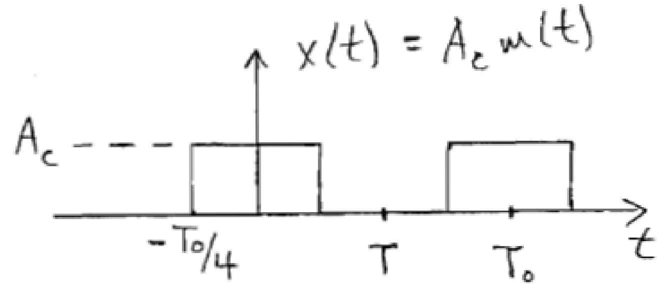
$$f_s = \frac{2}{T} = 2f_{IF}$$

$v_a(t)$ could be obtained by passing the output of the VCO (of the PLL) through a hard limiter.

5-46. (a.) $s(t) = x(t)\cos\omega_c t$, where

$T = \frac{1}{R} = \frac{1}{24000}$

$T_0 = 2T$



OOK:

$$S(f) = \frac{1}{2}\left[X(f-f_c) + X^*(-f-f_c)\right]$$

$$X(f) = \sum_{-\infty}^{\infty} c_n \delta(f-nf_0) \; ; \; x(t) = \sum_{-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} A_c e^{-jn\omega_0 t} dt = \frac{A_c}{T_0} \left.\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right|_{-T_0/4}^{T_0/4}$$

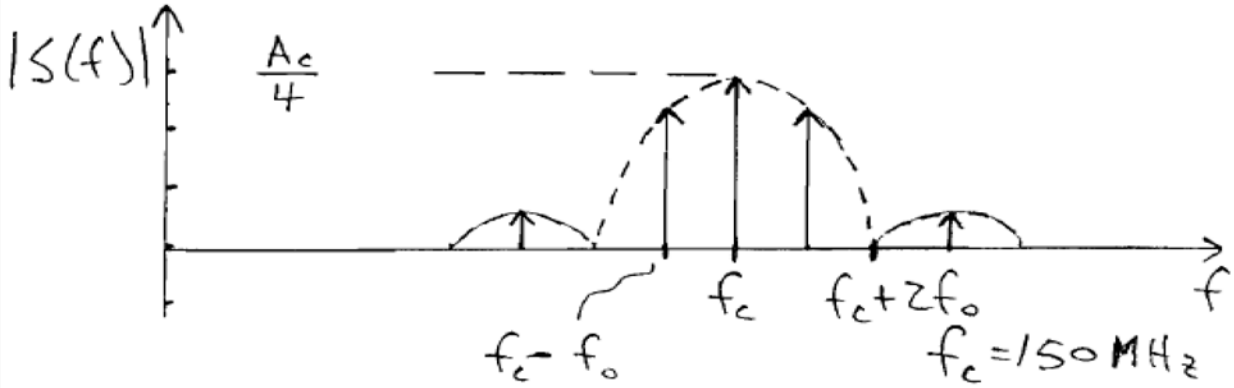
$$= \frac{A_c}{T_0} \frac{e^{-jn\pi/2} - e^{jn\pi/2}}{-jn2\pi/T_0} = \frac{A_c}{2} \frac{\sin(n\pi/2)}{n\pi/2}$$

$$X(f) = \frac{A_c}{2}\sum_{-\infty}^{\infty}\left[\frac{\sin(n\pi/2)}{n\pi/2}\right]\delta(f-nf_0)$$

$$f_0 = \frac{1}{T_0} = \frac{1}{2T} = \frac{R}{2}$$

$$S(f) = \frac{1}{2}\left[X(f-f_c) + X^*(-f-f_c)\right]$$

5-46. Cont'd (b.)



$f_c = 150$ MHz

$f_o = 12$ KHz

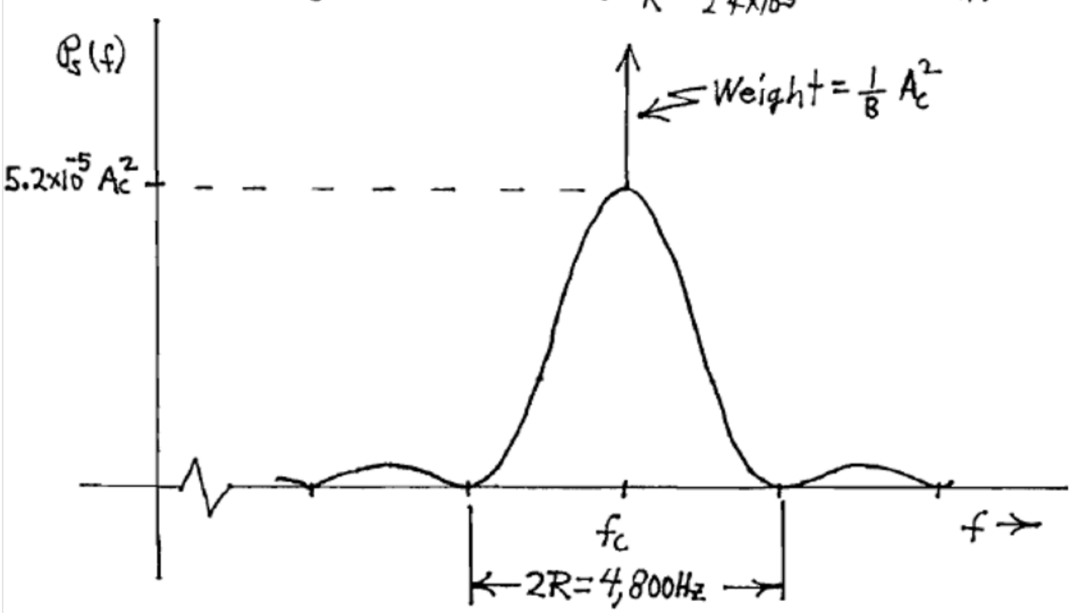
First zero-crossing at $n = 2$

$BW_T = 2(2f_o) = 2R = \underline{\underline{48 \text{ KHz}}}$

(c.) Using (5-2b) and (5-72)

$$\mathcal{P}_s(f) = \frac{1}{4}\frac{A_c^2}{2}\left[\delta(f-f_c) + T_b\left(\frac{\sin \pi(f-f_c)T_b}{\pi(f-f_c)T_b}\right)^2 + \delta(f+f_c) + T_b\left(\frac{\sin \pi(f+f_c)T_b}{\pi(f+f_c)T_b}\right)^2\right]$$

where $f_c = 150$ MHz and $T_b = \frac{1}{R} = \frac{1}{2.4 \times 10^3} = 0.0417$ msec



The null-to-null bandwidth is the same for both (b) and (c). Both have sinx/x type spectral envelope.

5-48.

$k := 0 \,..\, 7$ $\quad T := 0.0025$ $\quad t := 0, 0.00008 \,..\, 8\,T$

$a_k :=$

1
-1
-1
1
-1
1
1
-1

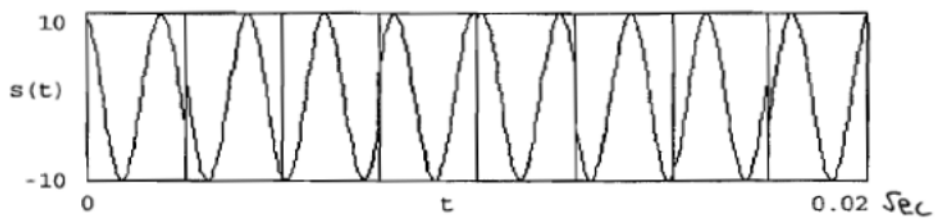
$\omega_c := 1000\,\pi$

$\mathrm{rect}(t) := \mathrm{if}\left[\left|t - \frac{T}{2}\right| \le \frac{T}{2}, 1, 0\right]$

$m(t) := \sum_k \left[a_k\,\mathrm{rect}(t - k\,T)\right]$

(a) SET INDEX $h := 0.2$ $\quad N := 64$ $\quad D_p := \frac{h\,\pi}{2}$ $\quad n := 1 \,..\, N - 1$ $\quad Ts := \frac{0.02}{N}$ $\quad j := 0 \,..\, \frac{N}{2}$ $\quad t1_n := n\,Ts$

$s(t) := 10\,\cos\left[\omega_c\,t + D_p\,m(t)\right]$



$M := 6$ $\quad N := 2^M$ $\quad N = 64$ $\quad k := 0 \,..\, N - 1$ $\quad T0 := 8\,T$

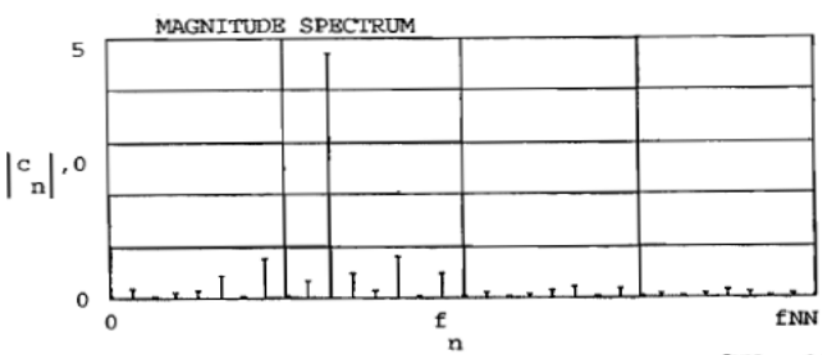
$dt := \frac{T0}{N}$ $\quad dt = 0$ $\quad t_k := k\,dt$

$ss_k := s(k \cdot dt)$ $\quad n := 0 \,..\, N - 1$ $\quad NN := 2^{M-1}$

Assume that the signal is periodic with period T0=8T. The spectrum can be obtained from the complex Fourier series coefficients. Furthermore, the complex Fourier series coefficients may be calculated using the FFT by substituting (2-178) into (2-186). Thus,

$c := \left[\frac{1}{\sqrt{N}}\right] \mathrm{icfft}(ss)$ $\quad fs := \frac{1}{dt}$ $\quad fs = 3200$ Hz

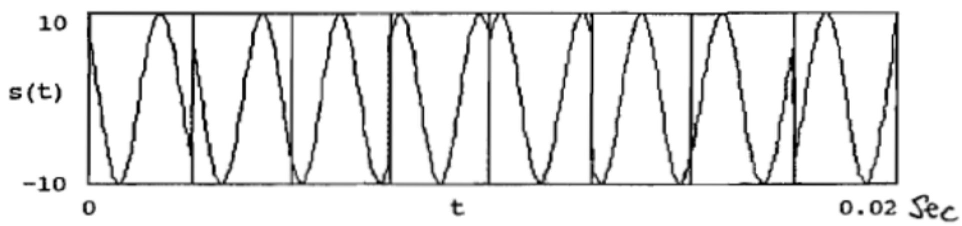
$f_n := \frac{n}{T0}$ $\quad f_1 = 50$ $\quad fNN := NN\,f_1$ $\quad fNN = 1600$

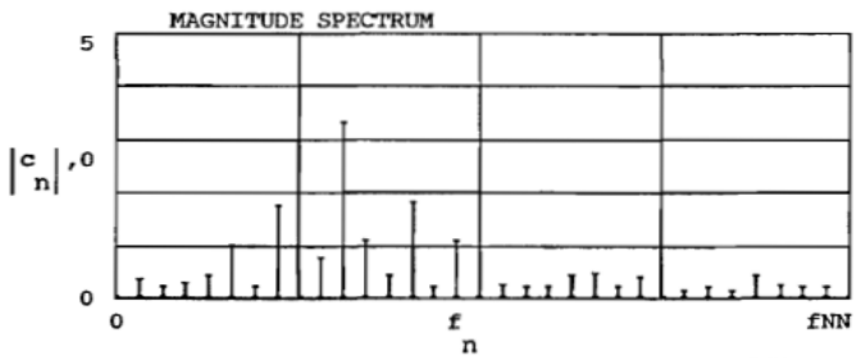


fNN = 1600 Hz

5-48. Cont'd

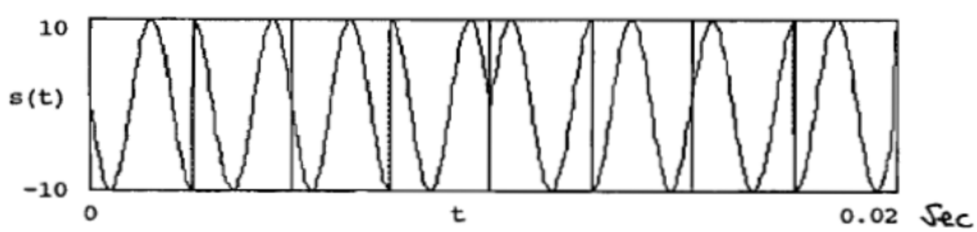
(b.) $h = 0.5$

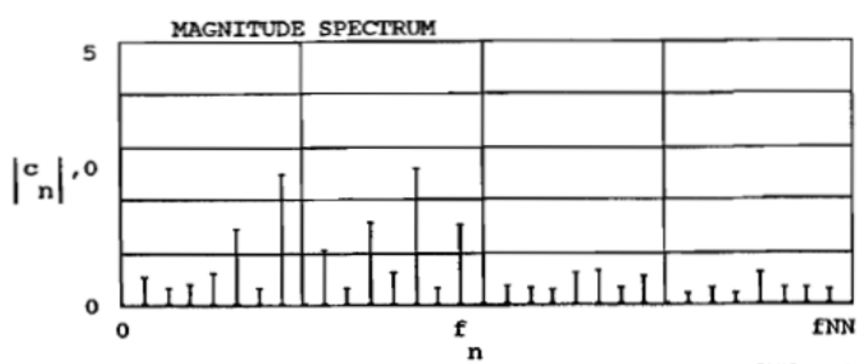




fNN = 1600 Hz

(c.) $h = 1.0$





fNN = 1600 Hz

5-53. $\Delta = R = \frac{2B}{1+r}$

(a.) OOK

(5-2) $B_T = 2B = (1+r)R = (1.5)\,64k$

$= \underline{\underline{96\text{ kHz}}}$

(b.) FSK

(5-7) $B_T = 2\Delta F + 2B$

$= 5\text{ kHz} + 96\text{ kHz} = \underline{\underline{101\text{ kHz}}}$

5-58. Use (5-106) $B_T = (1+r)\frac{R}{\ell}$ where $\ell = 2$ for QPSK

(a.) $\Rightarrow 24 = (1+r)\frac{30}{2} \Rightarrow (1+r) = \frac{2(24)}{30} = 1.6$

or $\underline{\underline{r = 0.6}}$

(b.) Max R allowed is when $r = 0$

$\Rightarrow R_{max} = \frac{2B_T}{1} = 2(24) = 48$ Mb/sec

$\Rightarrow$ No. A rolloff factor, r, could not be found support 50Mb/s QPSK signaling

$\boxed{5\text{-}59.}$

(a) Using the same procedure that leads to (5-102) where the FT of the square-root raised-cosine-rolloff pulse is obtained from (3-69)

$$F(f) = \sqrt{H_e(f)}$$

Then, using (6-70d) with $m_c = 0$ and $\sigma_c^2 = C$, the PSD of the complex envelope is

$$\mathcal{P}_g(f) = K|F(f)|^2 = K H_e(f)$$

$$\Rightarrow \quad \mathcal{P}_g(f) = K \begin{cases} 1, & |f| < f_1 \\ \frac{1}{2}\left\{1 + \cos\left[\frac{\pi(|f| - f_1)}{2f_\Delta}\right]\right\}, & f_1 < |f| < B \\ 0, & |f| > B \end{cases}$$

where $f_0 = \frac{1}{2\ell T_b} = \frac{1}{2}\left(\frac{R}{\ell}\right)$ and $\ell = 2$ for QPSK

$R = \frac{1}{T_b}$

5-59 a. Continued

$$f_1 = f_0 - f_\Delta = f_0 - r f_0 = f_0 (1-r) = \tfrac{1}{2}(1-r)\left(\tfrac{R}{\ell}\right)$$

$$f_\Delta = r f_0 = \tfrac{1}{2} r \left(\tfrac{R}{\ell}\right)$$

and

$$B = f_0 + f_\Delta = f_0 (1+r) = \tfrac{1}{2}(1+r)\left(\tfrac{R}{\ell}\right) \leftarrow \text{Absolute BW of complex envelope}$$

$$\mathcal{P}_v(f) = \tfrac{1}{4}\left[\mathcal{P}_g(f-f_c) + \mathcal{P}_g(-f-f_c)\right]$$

$\mathcal{P}_g(f) = \mathcal{P}_g(-f)$ for QPSK $\rightarrow$ $\mathcal{P}_g(f+f_c)$

The absolute bandpass bandwidth is

$$B_T = 2B = (1+r)\left(\tfrac{R}{\ell}\right) \text{ where } \ell = 2 \text{ for QPSK}$$

(b)

$r := 0.35$ <------ **Enter value of the rolloff factor, r**

$R := 1$ $\quad fo := 0.5 \cdot \left(\frac{R}{2}\right)$ $\quad N := 100$ $\quad n := 0, 1 .. N-1$

$df := 2 \cdot \frac{fo}{(N-1)}$ $\quad f_n := n \cdot df$

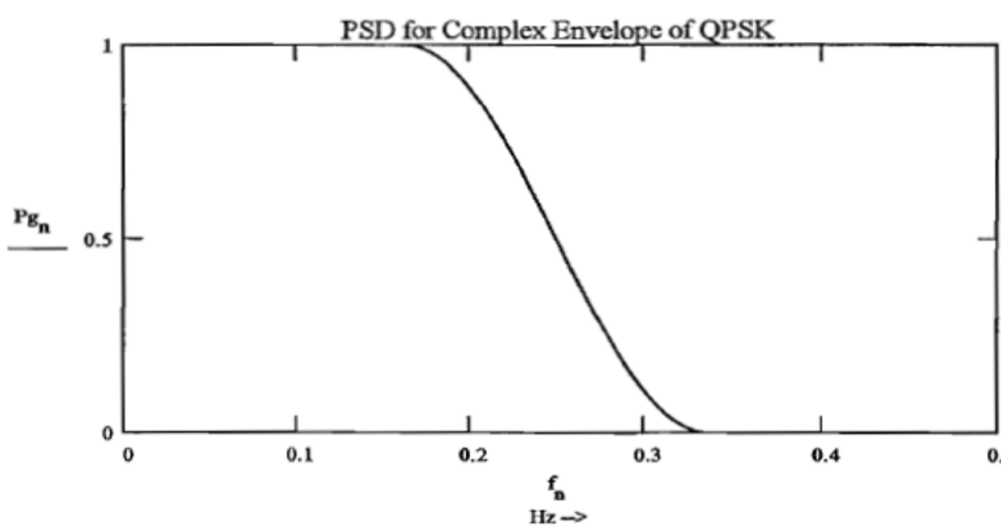
$fdel := r \cdot fo$ $\quad f1 := fo - fdel$ $\quad B := fo + fdel$

Construct the Raised Cosine frequency response using the Mathcad "if" function.

$$\cos1(f) := 0.5\left[1 + \cos\left[0.5 \cdot \pi \cdot \frac{(f - f1)}{fdel}\right]\right]$$

$Pg1(f) := if(f > f1, \cos1(f), 1)$ $\quad Pg(f) := if(f > B, 0, Pg1(f))$

$Pg_n := Pg(n \cdot df)$



5-67. From the description of $\pi/4$ QPSK in Sec. 5-10, use the table shown at the right

Input Bits	$\Delta\theta$
11	$+45°$
01	$+135°$
00	$-135°$
10	$-45°$

Data	10	11	01	00	10	10	10
$\Delta\theta$	$-45°$	$+45°$	$+135°$	$-135°$	$-45°$	$-45°$	$-45°$

(b) From (5-106)

$$B_T = \left(\frac{1+r}{\ell}\right)R \underset{\uparrow}{=} \left(\frac{1+0.5}{2}\right)R = \frac{\frac{3}{2}}{2}R = \frac{3}{4}R$$

$(r=0.5,\ \ell=2)$

$$B_T = \frac{3}{4}R \underset{\uparrow}{=} \frac{3}{4}(1.5) = \underline{\underline{1.13\ \text{Mb/sec}}}$$

$(R=1.5\ \text{Mb/s})$

5-71.

$i := 0\,..14$ $\quad n := 0\,..14$ $\quad t := -5, -4.99\,..5$ $\quad T := 1$ $\quad a_0 := -1$

$a_1 := -1$ $\quad a_2 := -1$ $\quad a_3 := 1$ $\quad a_4 := 1$ $\quad a_5 := 1$ $\quad a_6 := -1$

$a_7 := -1$ $\quad a_8 := -1$ $\quad a_9 := 1$ $\quad a_{10} := -1$ $\quad a_{11} := -1$ $\quad a_{12} := 1$

$a_{13} := -1$ $\quad a_{14} := -1$ $\quad tt := 0, 0.2\,..15$

$h(t) := \Phi(t) - \Phi(t-1)$ $\quad n1 := 0, 2\,..14$ $\quad n2 := 1, 3\,..13$

5-71. Cont'd.

$$hx(t) := \Phi(t) - \Phi(t-2)$$

$$m(tt) := \sum_{n} a_{n} \cdot h(tt - n\,T)$$

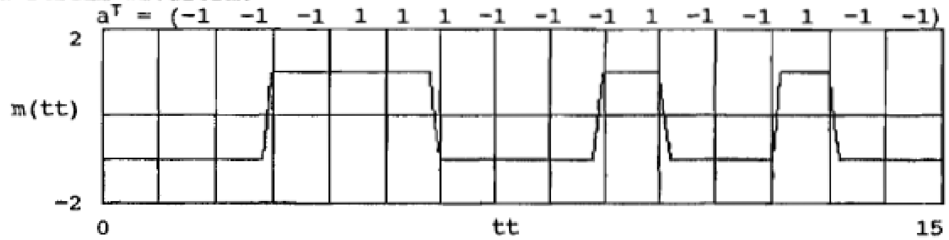
$$x(tt) := \sum_{n1} a_{n1} \cdot hx(tt - n1\,T) \qquad y(tt) := \sum_{n2} a_{n2}\, hx(tt - n2 \cdot T)$$

$$yy(tt) := \sum_{n2} a_{n2}\, hx(tt - n2 \cdot T)\, \sin\left[\pi\, \frac{tt - n2 \cdot T}{2}\right]$$

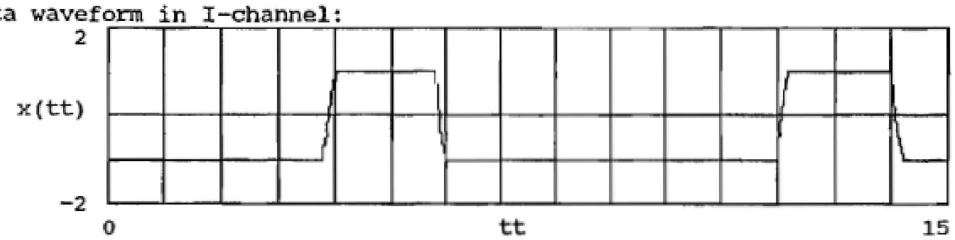
$$xx(tt) := \sum_{n1} a_{n1}\, hx(tt - n1\,T) \cdot \cos\left[\pi\, \frac{tt - n1\,T - 1}{2}\right]$$

Data stream waveform:

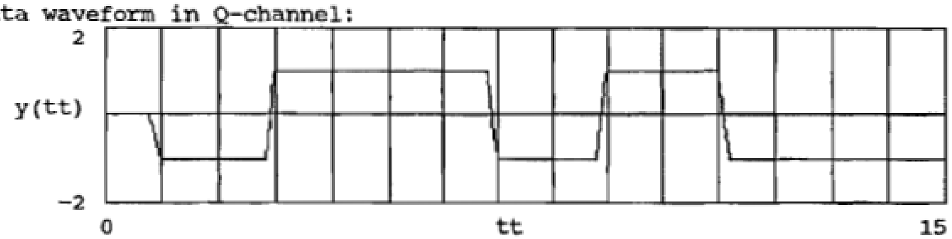
$a^T = (-1\ \ -1\ \ -1\ \ 1\ \ 1\ \ 1\ \ -1\ \ -1\ \ -1\ \ 1\ \ -1\ \ -1\ \ 1\ \ -1\ \ -1)$



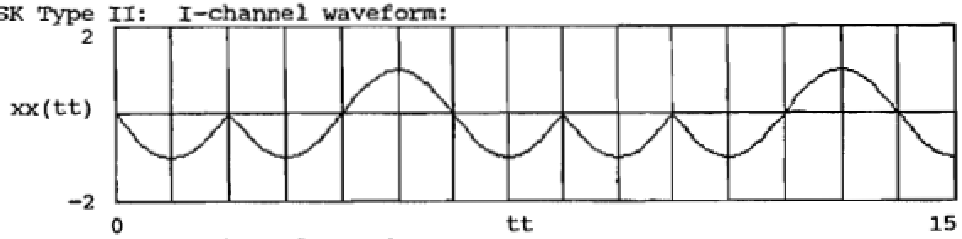
Data waveform in I-channel:



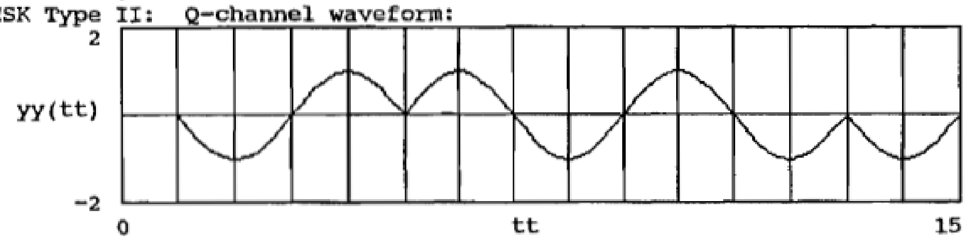
Data waveform in Q-channel:



MSK Type II: I-channel waveform:



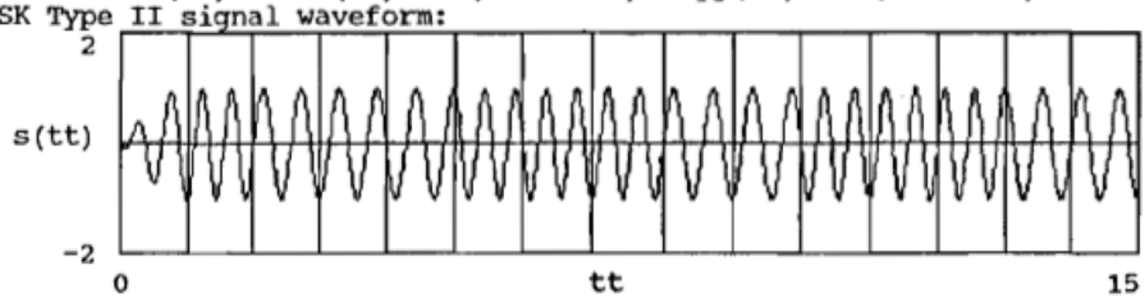
MSK Type II: Q-channel waveform:



5-71. Cont'd.

```
tt := 0,0.01 ..15
s(tt) := xx(tt) cos(2 π·2 tt) - yy(tt)·sin(2 π·2 tt)
```

MSK Type II signal waveform:



5-73. (a) $H(f) = e^{-\left(\frac{f}{B}\right)^2\left(\frac{\ln 2}{2}\right)} = \left[e^{-\pi\left(f\sqrt{\frac{\ln 2}{2}}\,\frac{1}{B}\,\frac{1}{\sqrt{\pi}}\right)^2}\right] \Rightarrow h(t) = B\sqrt{\frac{2\pi}{\ln 2}}\; e^{-\pi\left(tB\sqrt{\frac{2\pi}{\ln 2}}\right)^2}$

Table 2-2

$$\Pi\left(\frac{t}{T}\right) \longrightarrow \boxed{h(t)} \longrightarrow h_s(t)$$

$\Pi(-t) = \Pi(t)$

$$h_s(t) = \Pi\left(\frac{t}{T}\right) * h(t) = \int h(\lambda)\,\Pi\left(\frac{t-\lambda}{T}\right)d\lambda = \int h(\lambda)\,\Pi\left(\frac{\lambda-t}{T}\right)d\lambda$$

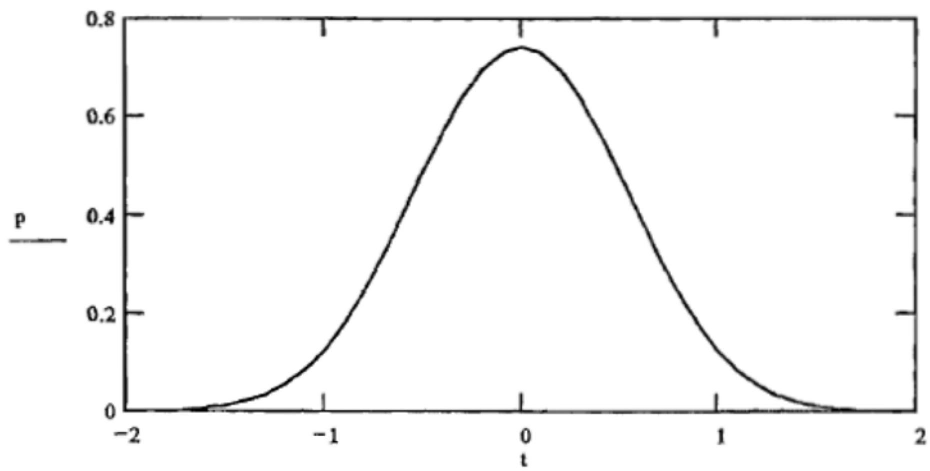
$$= \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \left(\sqrt{\frac{2\pi}{\ln 2}}\right) B\, e^{-\frac{2\pi^2}{\ln 2}\lambda^2 B^2}\, 1\, d\lambda$$

$$\Rightarrow h_s(t) = \left(\sqrt{\frac{2\pi}{\ln 2}}\right)(BT)\int_{\frac{t}{T}-\frac{1}{2}}^{\frac{t}{T}+\frac{1}{2}} e^{-\frac{2\pi^2}{\ln 2}(BT)^2 x^2}\,dx = p(t)$$

Let $\lambda = Tx$
$d\lambda = T\,dx$
$x = \frac{1}{T}\lambda$

(b) $BTb := 0.3 \quad Tb := 1 \quad N := 20 \quad dt := \frac{2\cdot Tb}{N} \quad n := 0, 1 .. 2\cdot N \quad t_n := (n - N)\cdot dt$

$$p(t) := \left[\sqrt{2\cdot\frac{\pi}{\ln(2)}}\cdot BTb\cdot\int_{\frac{t}{Tb}-0.5}^{\left(\frac{t}{Tb}\right)+0.5} e^{-\left[(2)\cdot\frac{(\pi^2)\cdot(BTb^2)\cdot x^2}{\ln(2)}\right]}dx\right] \qquad p_n := p(t_n)$$



$\boxed{5\text{-}79.}$

(a) Referring to Fig 5-42a, the FSK signal is

$v_1(t) = \cos[\omega_c t + \theta(t)]$ where $\theta(t) = D_f \int^t m(\lambda)\,d\lambda$

The output of the FH spreader is

$$v_2(t) = A_c \cos[\omega_c t + \theta(t)] \cos(\omega_i t)$$
$$= \frac{A_c}{2} \cos[(\omega_c - \omega_i)t + \theta(t)] + \frac{A_c}{2} \cos[(\omega_c + \omega_i)t + \theta(t)]$$

The output of the BPF is the sum frequency part of $v_2(t)$

$$\Rightarrow \underline{\underline{s(t) = \frac{A_c}{2} \cos[(\omega_c + \omega_i)t + \theta(t)]}}$$

(b) Referring to Fig 5-42b the signal out of the FH spreader is $v_5(t) = s(t)\, 2\cos(\omega_i t) = A_c \cos[(\omega_c + \omega_i)t + \theta(t)] \cos(\omega_i t)$

$$= \underbrace{\frac{A_c}{2} \cos[\omega_c t + \theta(t)]}_{\text{diff term}} + \underbrace{\frac{A_c}{2} \cos[(\omega_c + 2\omega_i)t + \theta(t)]}_{\text{sum term}}$$

$\Rightarrow$ The output of the BPF is $\underline{\underline{v_6(t) = \frac{A_c}{2} \cos[\omega_c t + \theta(t)]}}$ which is FSK.